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Experimental studies on the onset of thermohaline convection

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Measurements of the heat flux and temperature gradient in a layer of saline solution stably stratified by Soret diffusion and heated from below exhibit the onset of convection by the growth of very small oscillations which eventually trigger finite amplitude convection. The critical Rayleigh numbers and periods of oscillation are consistent with published theoretical calculations. Several nonlinear modes are observed, even for the same heat flux, near the point of onset.

1. Introduction

Several years ago some remarkable effects were observed in a Rayleigh–Bénard convection experiment on sea water (Caldwell 1970). Observations of the temperature gradient as a function of heat flux showed (i) a reversal in slope, (ii) history dependence, (iii) oscillations and (iv) critical Rayleigh numbers substantially larger than 1708. At the time a number of factors appeared to be possible causes: (i) the combined effect of non-uniformity of fluid properties and non-ideal boundary conditions as considered by Busse (1967), (ii) nonlinearity of the temperature distribution as considered by Krishnamurti (1968) and (iii) concentration gradients set up by thermal (Soret) diffusion. The significance of the third possibility was debated in that previous paper (hereafter called I). The concentration gradient set up by Soret diffusion would lead to a situation similar to that considered by Veronis (1965), if the sign of the Soret coefficient S_T were opposite to the normal one. Usually the solute is caused by thermal diffusion to flow from hot to cold. Such diffusion would be destabilizing, and so could not cause an increase in critical Rayleigh number. The only existing measurements at sea-water concentration (Chanu 1958; Price 1961; Richardson 1965) showed normal Soret flow at room temperatures in NaCl. Our anomalous results were found only at near-zero temperatures. In KCl, S_T was known to decrease very rapidly as the temperature was lowered, so it seemed possible that S_T might change sign between room temperature and the temperatures at which our effects were found. Lack of knowledge about S_T frustrated any calculation of the effect.

Despite recognition of thermal diffusion of salt as an influence, it was rejected as a major cause for the following reasons. (i) It was expected that if it did have an influence at low temperatures it should also have an influence at higher temperatures, where its direction was known to be destabilizing, and no such destabilization was observed. As Hurle & Jakeman (1971) later pointed out, the influence of the convection caused by a destabilizing salt gradient on the heat flux is so weak that it is unobservable in those experiments. (ii) The stabilization was well correlated with $\Delta\beta/\beta$, the fractional change in thermal expansion across the fluid layer. Apparently this was coincidental; in fact there seems to be a linear correlation between β and S_T (Caldwell 1973). Similar phenomena were observed by Legros, Rasse & Thomaes (1970) in water–methanol and water–ethanol mixtures.

Recently, convection in fluid mixtures and solutions has received a good deal of attention, partly because of experiments by physical chemists and partly because of recent discoveries (of 'microstructure') by oceanographers. Veronis (1965, 1968) has studied the onset of steady and oscillatory convection generated by infinitesimal perturbations, and has also done some calculations on the onset of finite amplitude modes, all with free-surface boundary conditions. Hurle & Jakeman (1971) assumed a salt distribution set up by thermal diffusion, and included the Soret effect in their perturbation equations as they calculated the onset of steady and oscillatory modes for both free and solid boundaries, for infinitesimal perturbations only. Legros, Platten & Poty (1972) considered a similar situation, but used a numerical method to calculate critical Rayleigh numbers for the growth of infinitesimal disturbances in dilute solutions with solid boundaries, including in their analysis thermal diffusion.

Experimentally, Shirtcliffe (1967, 1969) found that, in a tank stably stratified with sugar, heating from below produced the onset of oscillatory convection, which grew until it triggered a steady finite amplitude mode. His measurements of the critical Rayleigh number do not, however, agree with Veronis's calculation, possibly because his experimental conditions were quite different from those assumed by Veronis. Hurle & Jakeman (1971) also observed an oscillatory instability, in a methanol–water mixture, the concentration gradient being set up by Soret diffusion only. Platten & Chavepeyer (1972) observed regular oscillations in a water–ethanol mixture.

In order to answer some of the questions raised in I, to examine the nonlinear behaviour of the system more closely and to obtain some data with which to compare the theoretical calculations, further experiments were undertaken. These were of three kinds: (i) pure water was placed in the same apparatus and the bath temperature set near 4 °C to see the effect of large $\Delta\beta/\beta$ with no solute present, (ii) runs were made with 0·5 N-NaCl, a much simpler solution than sea water, and (iii) the Soret coefficient for the NaCl solution was determined.

2. The Soret effect

The flux–gradient relations for heat and salt in a solution may be formulated (De Groot & Mazur 1962, p. 276) as

$$\left.\begin{aligned} \mathbf{J}_Q &= -k\nabla T - \rho TC(\partial\mu/\partial C)\,D'\nabla C, \\ \mathbf{J}_C &= -\rho D\nabla C - \rho D S_T C(1-C)\,\nabla T. \end{aligned}\right\} \qquad (1)$$

Here $\mathbf{J}_Q$ is the heat flux, $\mathbf{J}_C$ the solute flux, k the thermal conductivity, ρ the density, μ the chemical potential of the solute, D the diffusivity, S_T the Soret or thermal diffusion coefficient and D' the DuFour coefficient. In liquids one can neglect the second term in $\mathbf{J}_Q$, the DuFour transport, because the Onsager relations show that it will be small compared with $k\nabla T$. The second term in $\mathbf{J}_C$, the Soret transport, can be significant, however. In a fluid mixture, whether of different gases, of liquids or of solute and solvent, a mass flux is caused by a thermal gradient. The physical basis for this is obscure and calculations of the effect in liquids have not been successful.

In a solution initially uniform the imposition of a thermal gradient causes the solute to migrate in a direction either parallel or anti-parallel to the thermal gradient. This transport continues until enough solvent has moved to set up a concentration gradient. Down-gradient diffusion will then oppose the Soret flux, and in a closed container an equilibrium between the fluxes will be reached, so that the solute distribution no longer changes. Concentration and temperature gradients are then related by

$$\nabla C = -S_T C(1-C)\nabla T, \tag{2}$$

assuming that the fluid is still.

Another paper (Caldwell 1973*a*) describes measurements of S_T and D in 0·5 N-NaCl, the solution used in the present work, which yield

$$\left.\begin{aligned} S_T &= 10^{-3}[-1{\cdot}232+0{\cdot}1128T-0{\cdot}00087T^2]\,^{\circ}\mathrm{C}^{-1}, \\ D &= 10^{-5}[0{\cdot}44+0{\cdot}0423T]\,\mathrm{cm}^2/\mathrm{s}, \end{aligned}\right\} \tag{3}$$

where T is the temperature in °C. The error in S_T is estimated to be $5\times10^{-5}\,\mathrm{cm}^2/\mathrm{s}$ and that in D to be $10^{-6}\,\mathrm{cm}^2/\mathrm{s}$.

Thus S_T does change sign, at about 12 °C. Below this temperature salt flows from cold to hot.

3. Previous work

In order to summarize the various theoretical calculations, it is convenient to settle first on a set of parameters to use in describing the system. We shall use R, the ordinary Rayleigh number, Pr, the Prandtl number ν/χ, and Sc, the Schmidt number ν/D. A way of expressing the degree of salt stratification is also required. We shall use

$$\gamma = \frac{\beta'}{\beta}\frac{\Delta C}{\Delta T}, \tag{4}$$

where β' is the saline contraction, β the thermal expansion coefficient and ΔC and ΔT the total changes in concentration and temperature, respectively.

If the salt distribution is set up by the Soret flux and if C, the mass fraction of salt, is small compared with unity (true for any salt solution short of a brine), then by (2)

$$\gamma = (\beta'/\beta)S_T C. \tag{5}$$

This is the same γ, then, as that used by Hurle & Jakeman (1971).

Veronis (1968) (and Shirtcliffe 1969) used a quantity called R_S, a solute Rayleigh number. For reasonably dilute salt solutions,

$$R_S = \gamma R, \quad \gamma = R_S/R. \tag{6}$$

Legros *et al.* (1972) use the quantities R_{TH} and $\mathscr{S}$, which are related to ours by

$$R_{TH}\mathscr{S} = \gamma R. \tag{7}$$

The parameter γ is also called the 'stability ratio' when applied to double-diffusive phenomena (Turner 1965).

One revealing comparison is that between the results of Veronis and of Hurle & Jakeman for free boundaries. Translating to our parameters, Veronis found for steady and overstable modes respectively

$$R_c^s = R_{FF}\Big/\left[1-\frac{Sc}{Pr}\gamma\right], \quad R_c^o = R_{FF}\Big/\left[1-\frac{Pr}{1+Pr}\gamma\right], \tag{8}$$

where $R_{FF} = \frac{27}{4}\pi^4$ and the superscripts s and o stand for 'steady' and 'oscillatory' respectively. Hurle & Jakeman found

$$R_c^s = R_{FF}\Big/\left[1-\gamma\left(1+\frac{Sc}{Pr}\right)\right], \quad R_c^o = R_{FF}\frac{(Sc+1)(Sc+Pr)(1+Pr)}{Sc^2[1+Pr(1-\gamma)]}. \tag{9}$$

The differences between the results are very small for salt solutions, for which $Pr \gg 1$ and $Sc \gg Pr$. Only when γ is very near zero is the difference significant: Veronis predicted $R_c^o < R_c^s$ for all $\gamma > 0$ whereas Hurle & Jakeman predicted $R_c^o > R_c^s$ for small positive values of γ ($< 10^{-4}$ for salt solutions). The difference between the two sets of results is never more than 1 % for salt solutions, experimentally undetectable. Hurle & Jakeman included the Soret flux in their equations but Veronis did not. Thus the Soret transport does not appreciably affect the dynamics of onset, but only sets up the initial stratification. However, we know that the diffusive salt transport plays little part in the dynamics, so we might have expected inclusion of the Soret flux to make little difference.

We shall concentrate on the oscillatory modes. They are the ones we expect to see because their R_c is much smaller.

Legros *et al.* (1972) found that there is a steady convection flow corresponding to oscillatory modes of zero frequency having lower R_c than the steady motions considered by Veronis. The truly oscillatory modes have still lower R_c, so they remain the ones of interest.

Does the nature of the boundaries matter? We can compare Hurle & Jakeman's results for free, perfectly conducting, permeable boundaries [equation (9)] with their computations for rigid, perfectly conducting, impermeable boundaries. Because R_c for the latter cannot be expressed as a formula the comparison is best made graphically (see figure 6), using the ratio of R_c at each γ to the appropriate R_c for $\gamma = 0$. We note that, for given R_c, the values of γ differ by less than 4 %. So boundary conditions make no great difference, especially for moderate values of γ.

Legros *et al.* (1972) considered a system with rigid, perfectly conducting, impermeable boundaries, also incorporating the Soret flux. If we divide their

R_c by their R_c for $\gamma = 0$, we find agreement with the free-boundary case, for the moderate $S_T \Delta T$ appropriate to salt solutions.

Finite amplitude modes. Veronis (1965, 1968) made two calculations for finite amplitude, concluding in the latter paper that infinitesimal perturbations will grow into oscillatory modes *before* finite amplitude motions can induce steady convection. These calculations assumed free, permeable boundaries and used a Schmidt number of 31·6, almost 100 times smaller than that in our experiments, so this conclusion may not be applicable.

Shirtcliffe (1969) heated the bottom of a tank of stably stratified sugar solution. In looking at his results we must remember that the experiment was transient, the temperature profile changing with time. Since the convective motions took place only in a thin layer at the bottom of the tank the upper boundary conditions were not well defined.

He found that convection was initially oscillatory and then grew to large amplitudes, which might have represented a finite amplitude mode. Determinations of R_c were complicated by the nonlinearity of the density profile. His values of R_c were much higher than expected.

Hurle & Jakeman (1971) performed an experiment with apparatus similar to ours, using water–methanol mixtures because some measurements of the Soret coefficients had been made in these solutions. The concentration gradient in these experiments was set up by Soret diffusion, but because of the high thermal expansion of their solutions, values of γ greater than 0·1 could not be achieved, so their results are more qualitative than quantitative. They also saw the onset as oscillatory. Like Shirtcliffe's, their experiment was transient and they could not maintain a constant heat flux while studying the development of the motion.

In summary, from the Veronis calculations it seems that convection should begin in our system in an oscillatory mode which may, when developed, trigger a steady finite amplitude mode. Finite amplitude calculations directly applicable to our case have not been done. In the previous experiments oscillatory motions were observed. In neither do the values of R_c give quantitative confirmation of the theories. Shirtcliffe found R_c which were too high, possibly because of the differences between his set-up and that assumed by the theory. Hurle & Jakeman could not achieve a large enough γ to obtain sufficient stabilization, in view of the scatter in their data, for quantitative comparison. Both found the periods of oscillation to be in some agreement with predicted values. The transient nature of these experiments made accurate determination of periods difficult.

4. Apparatus and procedure

The device as shown (figure 1) was modified very little from the description given in I, so the reader is referred to that paper for details. Basically, a fluid layer 0·635 cm deep is heated from below, while the top of the layer is maintained at a constant temperature. The heat flux is determined electrically and the temperatures of the top and bottom measured by means of thermistors.

The device was no longer in a pressure bomb, but was placed in a Neslab T 45 temperature bath and thermally lagged for some runs; later it was placed in

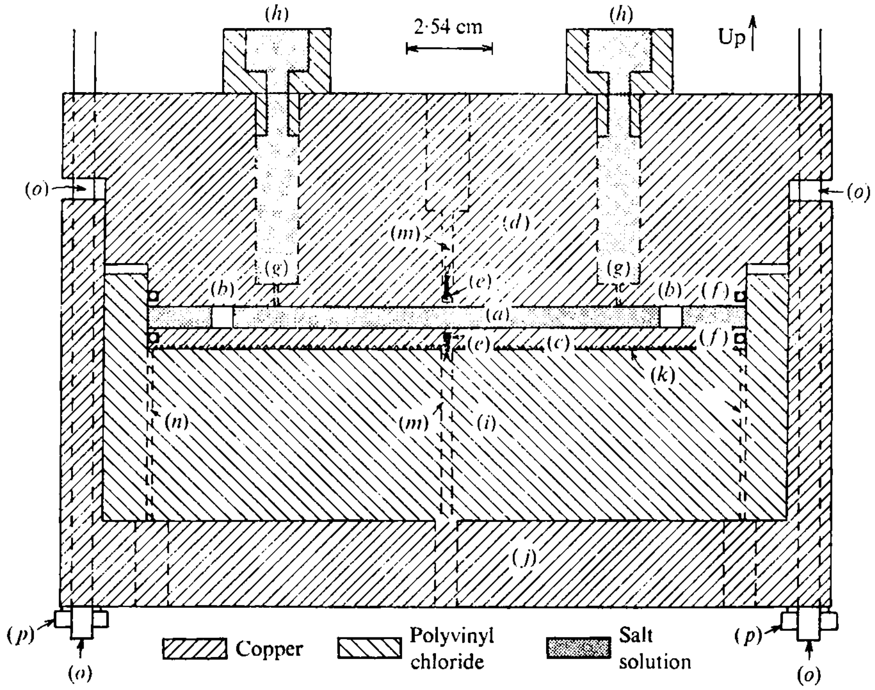

FIGURE 1. Section through the apparatus. This device should be imagined as immersed in hydraulic oil, inside a temperature bath. The top plate (*d*) and bottom plate (*c*) are separated by three (only two are shown) 0·635 cm high quartz blocks (*b*). Heat supplied by the resistance wire (*k*) flows through the fluid layer (*a*), plate (*d*), the cap [located above plate (*d*) but not shown here], and finally to an external water bath. O-rings (*f*) seal the fluid layer from the hydraulic oil. Polyvinyl chloride (*i*) insulates the fluid sample and bottom plate. Thermistors (*e*) measure plate temperatures. Reservoirs (*h*) supply or accept fluid through 0·08 cm holes (*g*) as changes in temperature require. Plate (*c*) is 17·8 cm in diameter and 0·635 cm thick. The insulation (*i*) has a 5·08 cm thick bottom and 1·27 cm thick walls.

a more precisely regulated bath described elsewhere (Caldwell 1973). Fluctuations in temperature measured with no heat flux were undetectable, being less than 10^{-4} °C. The procedure and calculations were also quite similar to those described in I except that much more time was taken over the runs. After each change in the heating rate we waited as long as was necessary for equilibrium, usually 12 h but sometimes days, as will be seen. Every measurement shown can be regarded as steady state.

The Rayleigh number was calculated from ΔT, the difference in temperature between the thermistors, using values of the various parameters appropriate to sea water of the same concentration (28·7 ‰):

$$R = g\beta\Delta T d^3/\nu\chi. \tag{10}$$

Here g is the acceleration due to gravity, β the thermal expansion coefficient, d the depth of the fluid layer, ν the kinematic viscosity and χ the thermal diffusivity. Sea-water values were used because better ones for NaCl are not available. The differences, where known, are not great. The values of the various properties

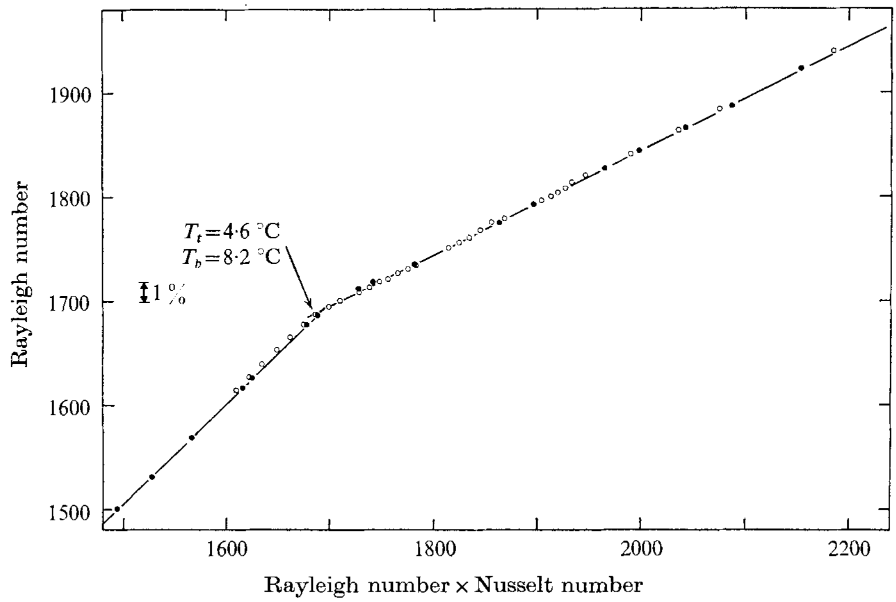

FIGURE 2. Non-dimensionalized ΔT *vs.* heat flux for a layer of pure water. At the critical point, the top of the layer is at T_t = 4·6 °C, the bottom at T_b = 8·2 °C, so the thermal expansion changes by 148 % across the layer. Still R_c is not significantly different from 1708, and no hysteresis is seen in the plot. ●, data taken after an increase in heat flux; ○, data taken after a decrease.

were taken as those at the mean temperature of the two plates. Concentration variations within the fluid were not sufficient to affect the values of these properties significantly. Although there is always some doubt about a calculated value of a Rayleigh number because of the variation in the values of the physical properties found in the literature, these calculations cannot be too much in error because the value calculated for the critical Rayleigh number at high temperatures, where it is expected to be 1708, is 1675, only 2 % out.

Nusselt numbers, *Nu*, were calculated by assuming *Nu* to be unity when convection was absent; that is, if for a given measurement the power input was *H* and temperature difference ΔT, and for a previous measurement when convection was absent the power input was H_{cond} and the temperature differential was ΔT_{cond}, then

$$Nu = (H/\Delta T)/(H_{\text{cond}}/\Delta T_{\text{cond}}). \tag{11}$$

5. Onset of convection in pure water near 4 °C

A number of runs in pure water failed to reveal any sign of the behaviour characteristic of sea water. For the run shown (figure 2) the temperature of the top of the fluid layer was 4·6 °C, and that of the bottom 8·2 °C at the onset of convection. Thus the relative change $\Delta\beta/\beta$ in thermal expansion across the layer was 1·48. No sign of hysteresis is seen, and the critical Rayleigh number is 1690, 1 % lower than predicted by the calculation for a pure fluid with constant properties.

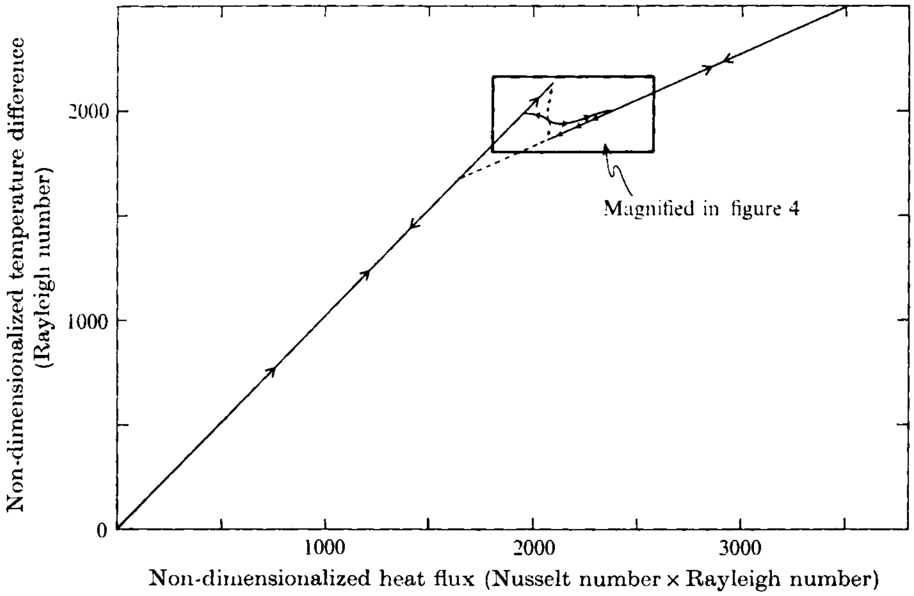

FIGURE 3. R *vs.* $R \times Nu$ for 0·5 N-NaCl: the measured points lie along the solid lines, but are too close together to represent on this scale. At the point of onset of convection, the mean temperature was 3·58 °C and the temperature difference was 2·34 °C.

Davis (1964) would predict a 7 % lowering of R_c for this value of $\Delta\beta/\beta$. In the previous experiments using sea water, the increase in R_c was definite for $\Delta\beta/\beta = 0{\cdot}05$. We must conclude that property variation is *not* the significant factor we had thought in I.

6. The heat-flux plot in a salt solution

In contrast to figure 2, when a 0·5 N-NaCl solution at a temperature of about 4 °C is placed in the same apparatus quite a remarkable heat-flow plot is obtained (figures 3, 4). In this case, instability occurred at a Rayleigh number of about 2140. The points just before onset were taken only daily because onset occurs so slowly. An increase in heat flux from just below critical to just above results in the onset of convective motions. After several days the system again reaches equilibrium, at a much lower Rayleigh number. The events occurring at onset will be discussed in a later section.

If after a convective equilibrium is attained the heat flux is next decreased by small amounts, the Rayleigh number remains fairly constant, increasing slightly until the conduction line is regained, at a Rayleigh number of about 1990.

If instead the next steps in heat flux are upward, we proceed to the right in figure 4, with the Rayleigh number changing little. At $R \times Nu \simeq 2400$ the plot becomes linear, and remains so, at least until $R = 4000$. If we next decrease the heat flux, the data points remain on the same line down to $R \simeq 1880$. The next step down in heat flux after that causes a sudden increase in R, to about

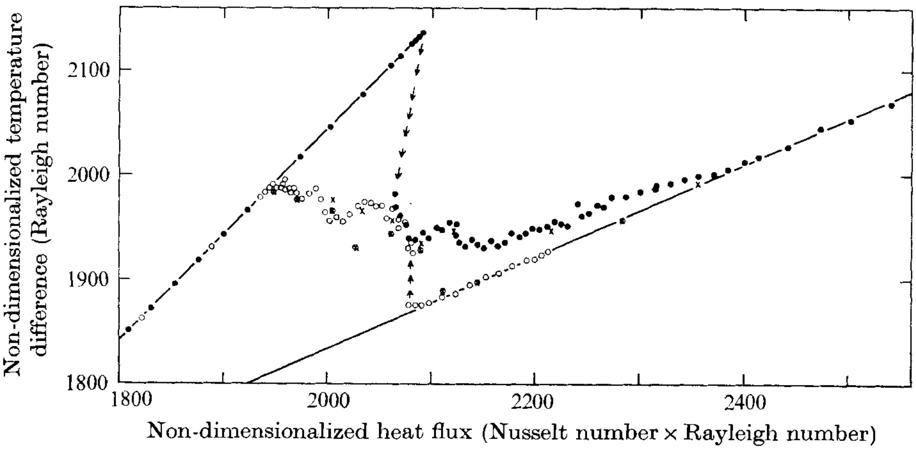

FIGURE 4. R vs. $R \times Nu$ for 0·5 N-NaCl: this figure covers the region outlined in figure 3. ●, measurements taken after small increases in heat flux; ○, measurements taken after small decreases; ×, heat flux suddenly increased from zero, then left steady until equilibrium; ⊗, heat flux decreased from maximum suddenly, then left until equilibrium.

1930. Continuing decreases eventually restore the conduction condition, the same path being followed as when conduction was regained by proceeding from the first convective point.

If we extrapolate the linear portion of the plot at high R, the conduction line is met at $R = 1675$, a value which is within 2 % of the critical Rayleigh number for the pure fluid, 1708.

There seem to be four distinct modes of heat transfer for the system: (i) conduction, (ii) an intermediate (oscillatory) mode where R varies only between 1930 and 2000 although $R \times Nu$ varies from 1940 to 2400, (iii) another oscillatory mode, existing for some of the same values of heat flux as (ii), but reached by decreasing the heat flux from large values, in which R depends linearly on $R \times Nu$ but the periods of oscillation are much longer, and (iv) 'fully developed' convection where $R \times Nu$ is a linear function of R. There exists a small range in $R \times Nu$ in which the system might be in any of the first three states (near $R \times Nu = 2080$).

An intuitive picture can be formed of the initial onset. When the system is in the conduction state, Soret diffusion stratifies the solution, making it more stable because the Soret flux is from cold to hot at this temperature. Thus greater buoyancy forces are required to force the fluid to move. As ΔT is increased, the buoyancy finally overcomes the stratification and some motion begins. This motion is much more effective in changing the salt distribution than in modifying the temperature distribution, because, since $D \ll \chi$, diffusion cannot resist the distortion of the salt distribution very well. As the salt is remixed, the resistance to the buoyancy forces decreases, and the convection becomes more intense. Finally an equilibrium (state (ii)) is reached. Now the salt can be unmixed again only by a return to conduction. State (ii) is then one in which the salt distribution is neither that of diffusive equilibrium, nor completely mixed. When the heat

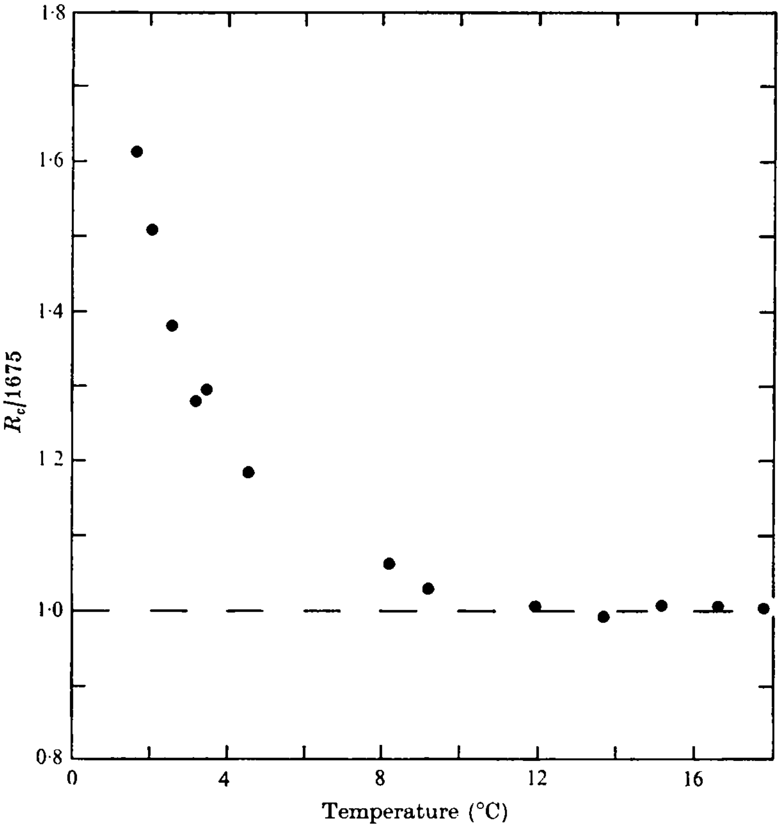

FIGURE 5. Critical Rayleigh number *vs.* mean temperature of sample, 0·5 N-NaCl. R_c is taken as the highest Rayleigh number reached before the sudden drop for the lower temperatures where the heat-flux plot resembles figure 4, but as the intersection of straight lines fitting measurements above and below the change in slope for the higher temperatures where the plot resembles figure 2.

flux is increased sufficiently, the salt does become completely mixed and the system behaves as though the fluid were pure (state (iv)). When the heat flux is again decreased, the fluid still behaves as though pure until these convective motions become too weak to resist the tendency of the Soret flux to re-stratify. Then state (ii) is regained.

At this point we might wonder what happens if, for example, the heat flux is suddenly increased from zero to a value characteristic of convection. What state will be reached? The answer is seen in figure 4. There is a jump to state (ii) if it exists for this heat flux, otherwise to one of the others. For a sudden decrease in heat flux from the highest value, the system chooses to stay in state (iv) or in (iii) if they exist for the new value of the heat flux, otherwise it goes to state (ii).

For different mean temperatures, different values of the critical Rayleigh numbers are calculated (figure 5). At low temperatures where the Soret flux is from cold to hot, R_c increases. At temperatures above 12° where the Soret flux is destabilizing, R_c becomes constant.

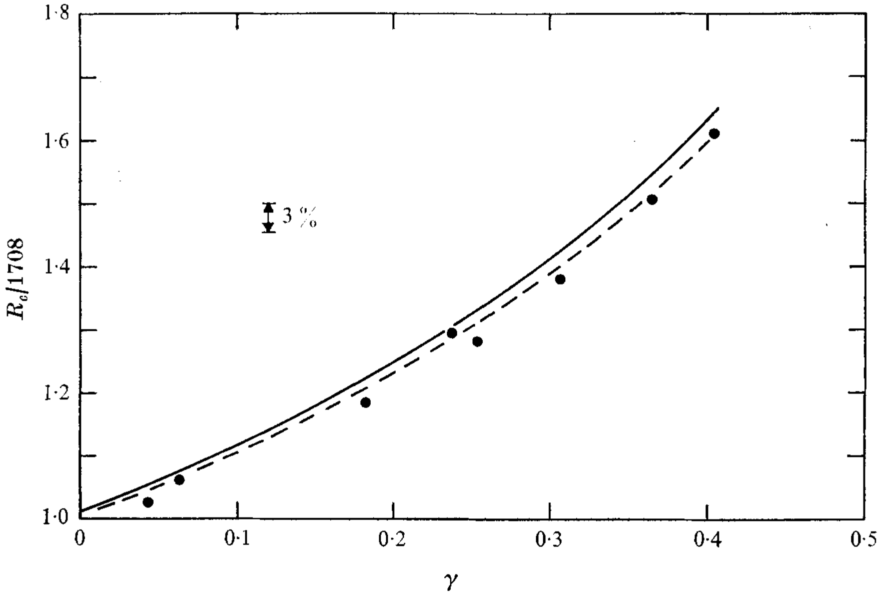

FIGURE 6. Theoretical and experimental values of R_c *vs.* stability ratio γ. For rigid boundaries $R_c/1708$ is plotted as indicated on the axis. For the free boundaries $R_c/(\frac{1}{27}\pi^4)$ is plotted. The experimental points are plotted as $R_c/1675$ in order to correct for miscalculation of R caused by inaccuracies in values of physical constants. That is, we are assuming that we measure R_c as 1675 at high temperatures rather than 1708 because incorrect values of β, for example, have been used in the calculation of R_c. ——, rigid, impermeable, infinite boundaries; – –, free, permeable, infinite boundaries; ●, experiment ($R_c/1675$).

When I was written, it seemed strange that R_c was not lowered in the experiments run at higher temperature. The explanation (as given by Hurle & Jakeman 1971) is that the topheavy salt distribution does initiate motion for $R \ll 1708$, but that when that motion begins it very efficiently mixes the salt, again because D is so small, thereby tending to destroy the buoyancy force driving it. An equilibrium is immediately reached, in contrast to the progressive destabilization of the salt profile we see at low temperatures. The intensity of motion required to mix the salt does not transport enough heat to make the change in Nusselt number noticeable. In other words, if the salt profile is initially stabilizing, any motion will tend to mix the fluid, thereby reducing the stabilization due to the salt profile; so no equilibrium is possible until the salt profile is nearly destroyed. If the salt profile is initially destabilizing any motion lessens that effect and equilibrium is immediate. A very careful search was made for any change in the slope of the heat-flux plot for R less than the critical value of 1670, but none could be observed. The Nusselt number is constant to within 0·1 % for $R < 1670$.

So for destabilizing Soret flux, as R moves towards 1708 the salt is completely mixed, but the temperature profile is still fairly linear, and the stage is set for ordinary Rayleigh–Jeffreys instability.

T (°C)	ΔT_c (°C)	R_c	$S_T \times 10^3$ (°C^{-1})	γ
1·76	4·18	2690	−1·04	0·405
2·10	3·64	2520	−1·00	0·365
2·72	2·92	2300	−0·93	0·307
3·33	2·42	2140	−0·87	0·253
3·58	2·34	2160	−0·84	0·238
4·78	1·76	1975	−0·71	0·183
8·28	1·02	1770	−0·36	0·064
9·25	0·88	1720	−0·26	0·043

TABLE 1. Values of the temperature differences at onset of convection ΔT_c, mean temperatures T, critical Rayleigh number R_c, Soret coefficient S_T, and stability ratio γ, for the data used in constructing figure 6.

7. Comparison of theory and experiment for critical Rayleigh number

Comparison of experimental values of R_c with Hurle & Jakeman's theoretical values in a plot *vs.* γ (figure 6) shows agreement within experimental error, although there seems to be a tendency for R_c to be a bit lower than predicted. Because of the slow growth of the instability, which we shall be examining, we tend to *over*estimate R_c, but by less than 1 %. We should remember here that γ is calculated from values of the Soret coefficient calculated by extrapolation from measurements at temperatures above 12 °C. An error of 10 % in γ would not be impossible. The experimental data upon which figure 6 is based are summarized in table 1. We have discussed the question of the observed *vs.* theoretical critical Rayleigh numbers in different terms in another paper (Caldwell 1973*b*), where the observations are viewed as measurements of the Soret coefficient and detailed comparisons are made.

8. The process of onset

After the experiments mentioned above had been completed, the apparatus was modified so that a thermistor could be placed in the fluid layer. The thermistor, a glass-covered bead, has a diameter of 0·15 cm and a response time in still water of less than 1 s. Its stem projects inwards from the side of the fluid layer about 3 cm, and it is closer to the bottom plate than to the top. We hoped that we would see with it thermal oscillations suppressed at the thermistors in the end plates by thermal lagging.

The results near onset are quite striking (figure 7). In this case the heating power was increased from just below critical to just above, changing the Rayleigh number from 2135 to 2151. We see oscillations in the temperature, at first less than 0·001 °C in amplitude. After 3½ h they are well above noise, and are growing in amplitude. During this time the heat flux and ∇T remain constant within observational resolution. The period stays constant but the amplitude varies, although tending to increase. The short-term variations in amplitude superimposed on the growing trend might be caused by the movement of the

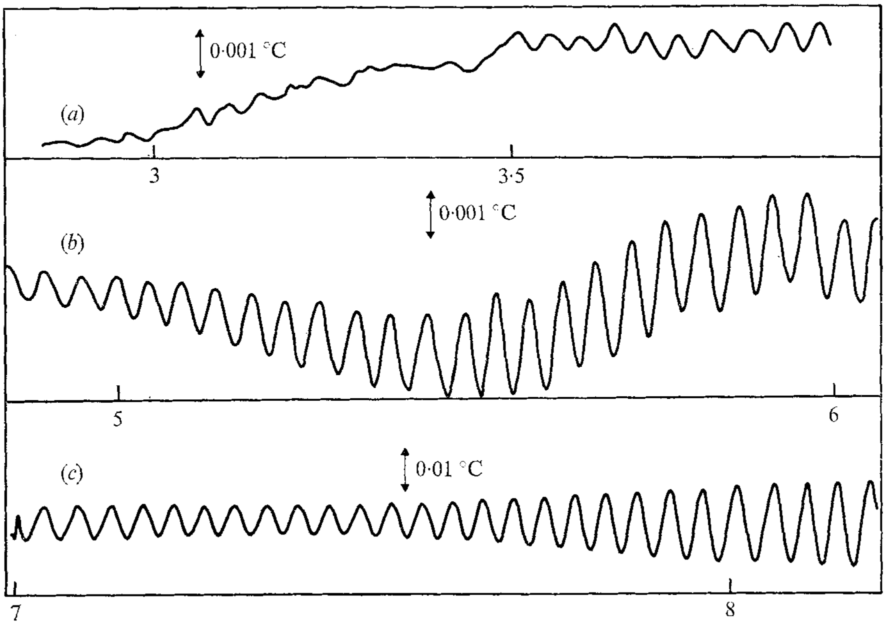

FIGURE 7. Temperature of thermistor inserted in fluid layer *vs.* time after step increase in heating rate past critical. Note that recordings (*a*), (*b*) and (*c*) are not continuous in time and that the scale in temperature changes between (*b*) and (*c*).

convection pattern relative to the thermistor. When the amplitude reaches a certain point, the period of the oscillations suddenly begins to increase and at about the same time the plate temperatures, which have been indicating a conducting state, start to change. A plot of the amplitude of oscillation, period and Nusselt number against time (figure 8) for another run, where the heat flux was increased by only 0·14 %, shows these effects. The amplitude grows exponentially, whereas the period and the Nusselt number change suddenly. In this case the oscillations grow very slowly, increasing by a factor of ten in about 35 h. After some time a quasi-steady state is reached, so that the temperature in the interior still oscillates, but with about twice the period of the original small oscillations. The system has dropped from conduction into state (ii).

More experiments of the same type were run to see what effect the size of the jump in heat flux had on the process of onset. Figure 9 shows a similar plot for a much larger jump in heat flux, about 1·5 %. The features are the same, but the time scale is greatly changed. Now the oscillations increase by ten times in only 50 min and the whole process is over in less than a day. The time required for the Nusselt number to change is about the same in each case, 90 % of the change being accomplished in about 12 h.

In figure 10 the growth rate of the oscillation is plotted against the highest Rayleigh number attained in each run. The bigger the increase in heat flux, the faster the growth rate. We might expect that for Rayleigh numbers below 2141 perturbations are damped.

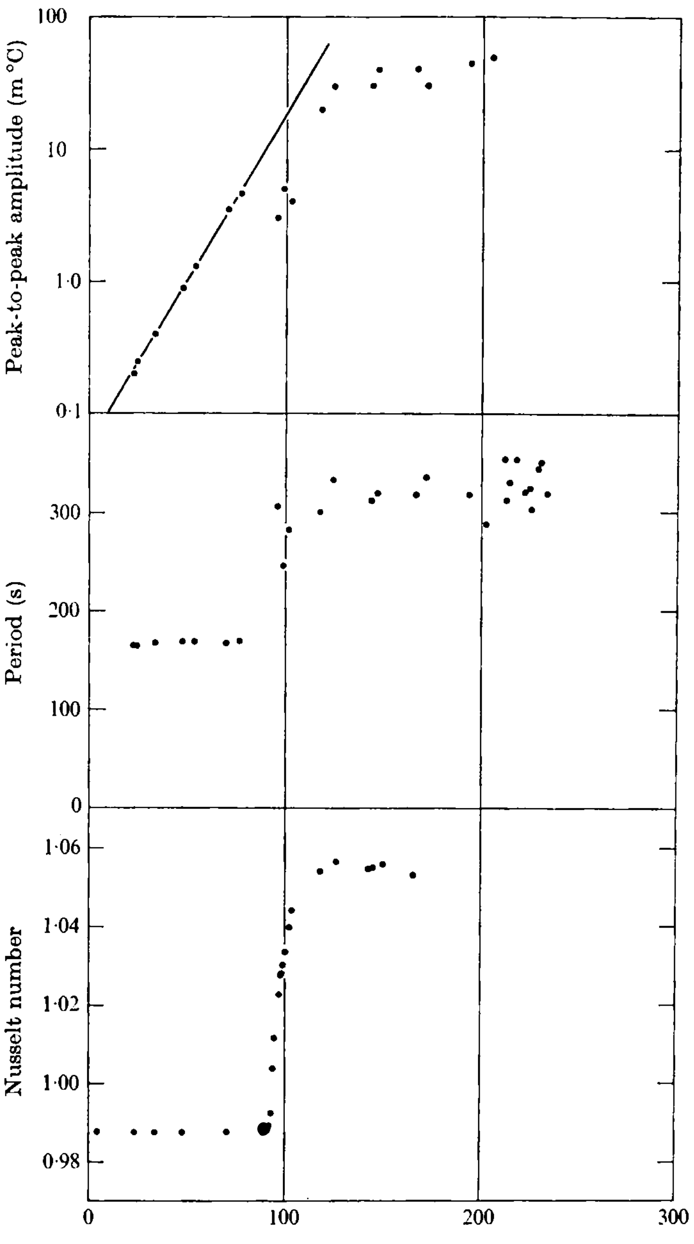

FIGURE 8. Amplitude of oscillations, period of oscillations and Nusselt number *vs.* time after step change in heating rate past critical. The Nusselt number is computed from the plate temperatures. At zero time the heat was increased by 0·14 %.

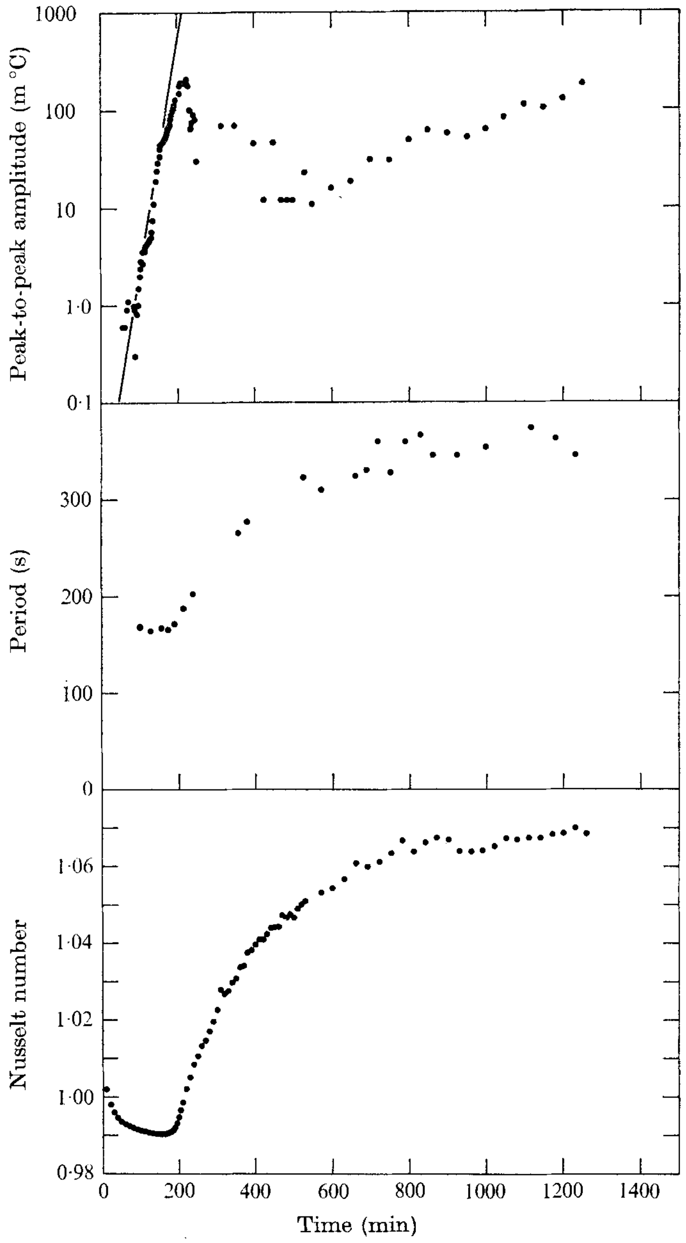

FIGURE 9. Amplitude of oscillations, period of oscillation and Nusselt number *vs.* time after sudden increase in heat by 2·1 %.

The periods of the small initial oscillation average 166 s, so the non-dimensional frequency $\sigma = 2\pi d^2/(\nu \times \text{period})$ is $0{\cdot}91 \pm 0{\cdot}02$. Calculations from Hurle & Jakeman's (1971) theory predict 0·90.

One way of looking at the small oscillations is in terms of the movement of the isothermal surfaces. Since the temperature in the fluid layer changes by

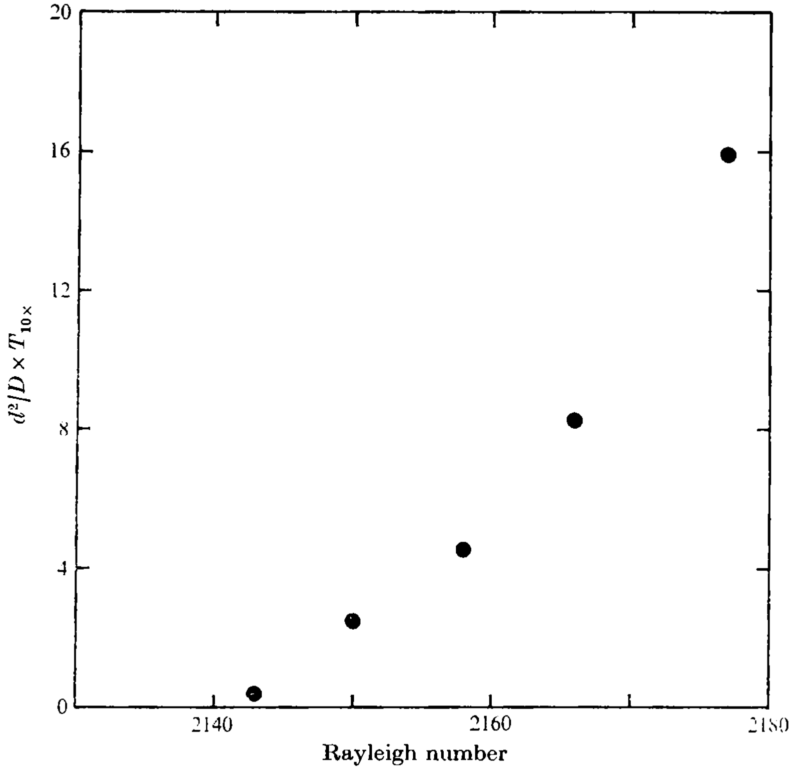

FIGURE 10. Non-dimensionalized growth rate of oscillations *vs.* Rayleigh number. $T_{10\times}$ is the time required for oscillations to increase by a factor of 10, d is the layer depth, 0·635 cm, and D is the salt diffusivity, $0{\cdot}6 \times 10^{-5}$ cm^2/s.

about 2·2 °C in 0·635 cm, a change in temperature of 0·001 °C implies an isotherm movement of 0·0003 cm. The oscillation amplitude characteristic of the time when the Nusselt number begins to increase, 0·02 °C, still corresponds to only a 0·006 cm movement of the isotherms. The period being 166 s, the vertical velocities must range from 2×10^{-6} cm/s for a 0·001 °C oscillation to 4×10^{-5} cm/s for a 0·02 °C oscillation: of the general order of 1 cm per day.

It seems safe to conclude that we are indeed seeing the growth of very tiny oscillations which finally become large enough to trigger the finite amplitude mode. Calculations by Velarde & Schechter (1972) show that such modes, when averaged in the horizontal, do not carry any net heat flux, so the Nusselt number should remain constant until the nonlinear mode is stimulated, just as observed.

Now we can ask what error was introduced by increasing the heat flux too fast when measuring R_c. We changed the heat flux by about 0·3 % per day, and so would skip too fast over the lowest Rayleigh number where convection can develop. Still, from these measurements we see that the error could not be as much as 1 %.

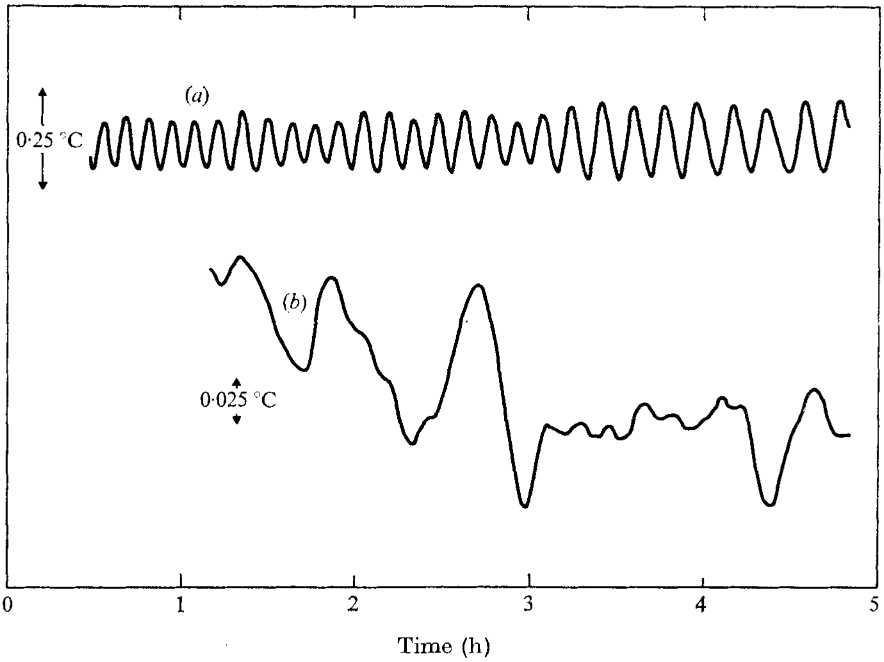

FIGURE 11. Temperatures indicated by thermistors (*a*) in the interior of the fluid layer and (*b*) in the bottom plate *vs*. time. The records were taken often waiting many days for 'equilibrium' after the onset of convection. This is state (ii).

9. Description of the nonlinear regimes

After the initial onset the system seems eventually to reach a quasi-steady state. Oscillations are seen in the interior of the fluid (figure 11) and the temperature of the lower surface wanders a bit. These oscillations have amplitudes, peak-to-peak, about 10 % of the temperature difference across the layer and have periods twice those of the small oscillations we saw at onset, for the same heat flux. The periods vary, perhaps because of movement of the convecting pattern past the thermistor. The temperature of the lower plate changes on a longer time scale, with an amplitude somewhat less. These changes persist indefinitely, and account for the scatter in figure 4.

The period of the oscillations depends on the heat flux. For example, a similar record taken for a lower heat flux (figure 12) shows shorter-period oscillations of smaller amplitude. The period of oscillation is shown plotted against Rayleigh number × Nusselt number in figure 13. Because of this variation in the period, perhaps we should resist the temptation to ascribe significance to the fact that the period seems to double during onset.

When we have increased the heat flux enough to reach the straight-line region (state (iv)), we find the oscillations gone and the plate temperatures quite steady. When, however, the heat flux is again decreased to state (iii) conditions we find oscillations (figure 14), but now of about the same period in both the interior of the fluid and in the bottom plate, although much reduced in amplitude

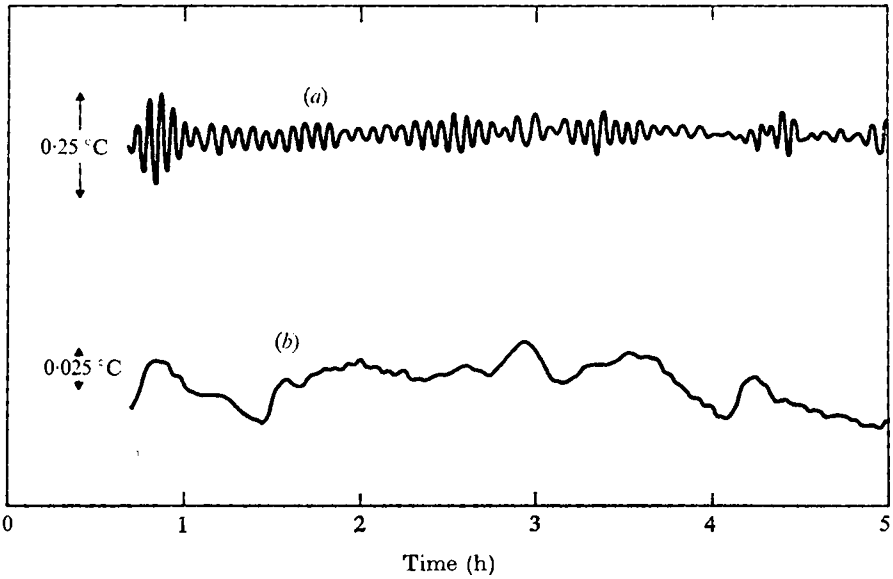

FIGURE 12. Temperatures indicated by thermistors (*a*) in the interior of the fluid and (*b*) in the bottom plate *vs.* time. The heat flux has been reduced somewhat; but the system is still in state (ii). The modulation seen in (*a*) is typical, possibly being caused by movement of the convection cells relative to the thermistor.

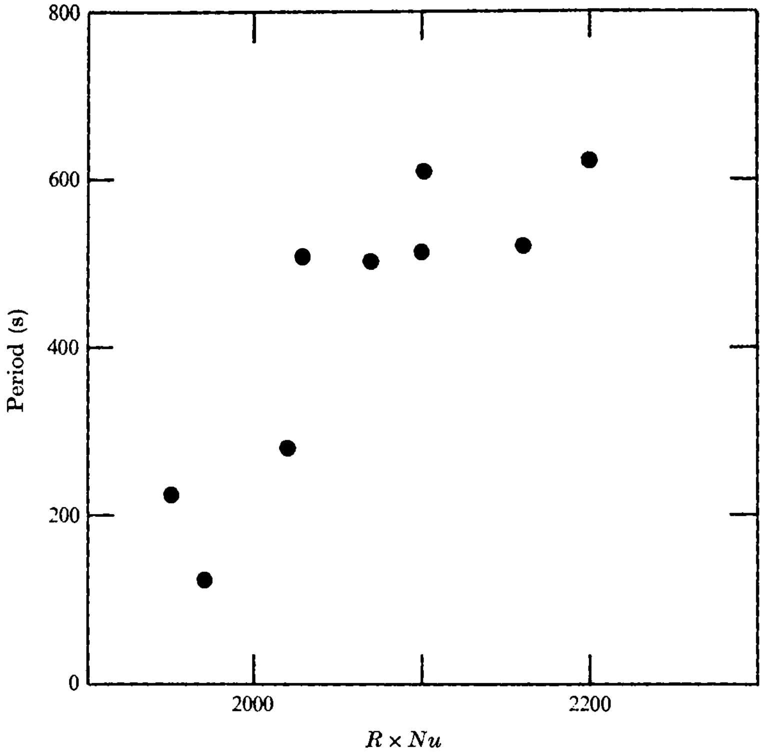

FIGURE 13. Period of oscillation *vs.* $R \times Nu$ in state (ii).

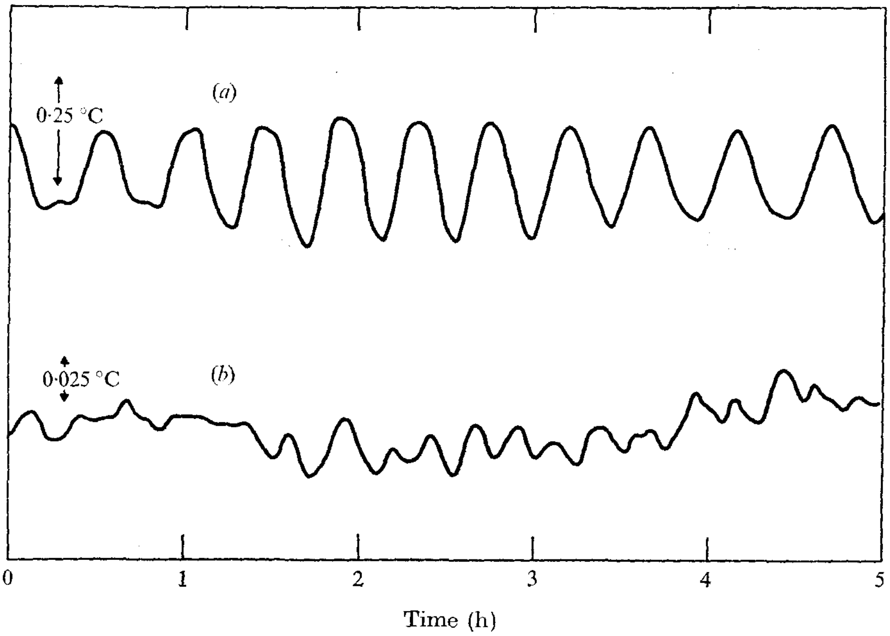

FIGURE 14. Temperatures indicated by thermistors (*a*) in the interior of the fluid and (*b*) in the bottom plate *vs.* time in state (iii). The oscillations in the interior seem coherent with those in the plate, and the period is not far from the thermal relaxation time of the plate.

in the plate. These may well be associated with the thermal capacity of the plate: it takes 30 min for the temperature of the plate to reach to 90 % of the final response, for a step change in heat flux in the conducting regime. The period, averaged over many runs, is about 27 min. Busse (1967) discussed such motions in a case such as ours where the heat flux is fixed and the solid layers have thermal capacity. We see little variation in period or amplitude of these oscillations with heat flux.

One question that has been raised is whether the large amplitude oscillations in state (ii) are caused by the Soret effect. Hurle & Jakeman (1973) found oscillations when their cell was not horizontal, when horizontal heat flux was present, and possibly also owing to the presence of the thermistor in the fluid, even for pure water at times. Our cell was carefully levelled and horizontal heat fluxes were eliminated as far as possible, but of course the oscillations were observed only when the thermistor was present. We cannot resolve this point, at present.

Another interesting question is to what extent the solution is acting as though homogeneous at high heat fluxes (state (iv)). In the experiments described here the extrapolation of R backwards from (iv) always intersected the conduction line near $R = 1708$, and the values of Nu were the same, within experimental error, as for pure water, suggesting that the fluid was acting as though homogeneous. One might expect that the fluid would never become completely homogeneous because a boundary layer near the plates is expected, even for large heat fluxes, where molecular diffusion dominates. For impermeable boundaries the salt flux must vanish at the plates (the normal component of $\mathbf{J}_C$ is zero) so, from (2), $\partial C/\partial n = -S_T C(1-C)\,\partial T/\partial n$. Thus there will always be a gradient of salt concentration at the boundaries. This gradient may affect the boundary layers even for very large Rayleigh number. On the other hand, if the boundary layers are being destroyed by 'thermal bursts', then there might not be time for re-stratification to occur between bursts, and the fluid might remain homogeneous. In these experiments sufficiently large Rayleigh numbers were not attained for any definite conclusions to be reached on this point. Further experiments with thicker layers of solution are now being performed to resolve this question.

10. Conclusions

(*a*) The onset of convection induced by heating from below in a stably salt-stratified solution does occur as the growth of very small amplitude oscillatory perturbations.

(*b*) The period of the oscillations remains constant as they grow, and has the value predicted by Hurle & Jakeman (1971) (independent of Rayleigh number).

(*c*) The growth is approximately exponential, with the growth rate depending on the Rayleigh number.

(*d*) After the oscillations have grown much larger, they suddenly change in character, their periods changing with time.

(*e*) When the period of oscillation changes, the Nusselt number, which has so far remained unity, increases greatly, reaching a new value in a time independent of the Rayleigh number.

(*f*) There are four possible modes of the system, for Rayleigh numbers less than 6000: (i) no convection, (ii) finite amplitude oscillatory convection, with period dependent on the heat flux, (iii) probably mixed modes, with oscillations associated with the thermal inertia of the boundary, and (iv) steady convection, which takes place at the highest Rayleigh numbers observed here.

(*g*) The critical Rayleigh numbers observed are in fair agreement with all theoretical calculations.

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